

APPENDIX

Calculation Strategies

Numeracy Skill by Year Group

Calculation Strategies

Addition & Subtraction

Below, you will find examples of problems involving the four operators, how they are taught, and approached by pupils at different stages in their school lives. **It is important to remember that, pupils on entry to secondary school may still be using these earlier methods of calculation.**

Pupils are encouraged to use mental calculations to support their written recordings.

Year	Addition	Subtraction												
3/4	<p>Example 1: $29 + 39 = 30 + 40 - 2$ $= 70 - 2$ $= 68$</p> <p>or</p> <p>$29 + 39 = 20 + 30 + 9 + 9$ $= 50 + 18$ $= 50 + 10 + 8$ $= 60 + 8$ $= 68$</p>	<p>Example 1: $90 - 37 = 90 - 40 + 3$ $= 53$</p> <p>or</p> <div style="text-align: center;"> <p>$\therefore 90 - 37 = 53$</p> </div>												
	<p>Example 2: $287 + 45$</p> $\begin{array}{r} 200 + 80 + 7 \\ + \quad 40 + 5 \\ \hline 200 + 120 + 12 = 332 \end{array}$ <p>or</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td style="text-align: right;">287</td><td style="text-align: right;">287</td></tr> <tr><td style="text-align: right;">+45</td><td style="text-align: right;">+45</td></tr> <tr><td style="text-align: right;">200</td><td style="text-align: right;">12</td></tr> <tr><td style="text-align: right;">120</td><td style="text-align: right;">120</td></tr> <tr><td style="text-align: right;">12</td><td style="text-align: right;">200</td></tr> <tr><td style="text-align: right;"><u>332</u></td><td style="text-align: right;"><u>332</u></td></tr> </table>	287	287	+45	+45	200	12	120	120	12	200	<u>332</u>	<u>332</u>	<p>Example 2: $567 - 243$</p> $\begin{array}{r} 500 + 60 + 7 - \\ \underline{200 + 40 + 3} \\ 300 + 20 + 4 = 324 \end{array}$ <p>or</p> <p>$767 - 619$ By exchanging a ten for ten units</p> $\begin{array}{r} 700 + 60 + 17 \\ \underline{600 + 10 + 9} \\ 100 + 40 + 8 = 48 \end{array}$
287	287													
+45	+45													
200	12													
120	120													
12	200													
<u>332</u>	<u>332</u>													
5/6	<p>Example: $8642 + 753$</p> $\begin{array}{r} 8000 + 600 + 40 + 2 \\ \quad \quad \quad 700 + 50 + 3 \\ \hline 8000 + 1300 + 90 + 5 \\ = 9395 \end{array}$ <p>or</p> <p>$8000 + (600 + 700) + (40 + 50) + (2 + 3)$ $= 8000 + 1300 + 90 + 5$ $= 9395$</p>	<p>Example: $2410 - 482$</p> $\begin{array}{r} \overset{1000}{2000} + \overset{1300}{400} + \overset{100}{10} + 0 \\ \underline{\quad \quad 400 + 80 + 2} \\ 1000 + 900 + 20 + 8 \\ = 1928 \end{array}$ <p>or</p> <p>2410 $\underline{-482}$ Using what they know about negative numbers -2 -70 $\underline{2000}$ 1928</p>												
7/8	<p>or</p> <p>8642 By this time they understand carrying</p> $\begin{array}{r} +753 \\ \hline 9395 \\ 1 \end{array}$	<p>or</p> <p>2410 $\underline{-482}$ By now they understand borrowing -2 -70 $\underline{2000}$ 1928</p>												

By Year 5/6 pupils will be able to extend their experience of addition and subtraction into a range of contexts, including calculations with money and measurement with decimals.

Multiplication and Division

Multiplication is usually associated with the idea of repeated addition, e.g.:

$$7 \times 6 = 6 + 6 + 6 + 6 + 6 + 6 + 6 = 42$$

Division is associated with repeated subtraction or sharing, e.g.:

$$42 \div 7 = 42 - 7 - 7 - 7 - 7 - 7 - 7 - 7 = 0 \quad (42 \text{ shared equally } 6 \text{ times})$$

Year	Multiplication	Division									
3/4	<p>Pupils will be taught multiplication tables up to 10 x 10 and associated facts, e.g.:</p> <p>If $7 \times 9 = 63$ then $9 \times 7 = 63$ $63 \div 9 = 7$ $63 \div 7 = 9$</p> <p>$4 \checkmark 4 + 4 \checkmark 4 + 4 \checkmark 4 = 6 \times 4$</p> <p>$8 + 8 + 8 = 8 \times 3$</p> <p>$\therefore 6 \times 4 = 24$ $8 \times 3 = 24$</p> <p>$7 \times 29 = (7 \times 30) - (7 \times 1)$ $= (7 \times 3 \times 10) - (7 \times 1)$ $= 210 - 7$ $= 203$</p> <p>$20 \times 30 = (2 \times 10) \times (3 \times 10)$ Using multiples = $2 \times 3 \times 10 \times 10$ of 10 = $6 \times 10 \times 10$ = 600</p>	<p>Recognition that division is the inverse of multiplication, e.g.:</p> <p style="text-align: center;">$63 \div 9 = 7$ became $7 \times 9 = 63$</p>									
5/6	<p>Example: 275×8 $(200 \times 8) + (70 \times 8) + (5 \times 8)$ $1600 + 560 + 40$ = 2200</p> <p>or</p> <p>$275 \times 2 = 550$ $275 \times 4 = 1100$ (doubled) $275 \times 8 = 2200$ (doubled)</p> <p>or</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td> <td>200</td> <td>70</td> <td>5</td> <td rowspan="2">Grid method</td> </tr> <tr> <td>8</td> <td>1600</td> <td>560</td> <td>40</td> </tr> </table> <p>= $1600 + 560 + 40$ = 2200</p>	x	200	70	5	Grid method	8	1600	560	40	<p>Working on the idea that division is repeated subtraction, e.g.:</p> <p style="text-align: center;">$458 \div 3$</p> <p>Since $3 \times 100 = 300$ and $3 \times 200 = 600$ the answer must be between 100 and 200 (estimation).</p> <p>$3 \times 100 = 300$ $2 \times 50 = 150$ (halving) $3 \times 150 = 450$ $\underline{3 \times 2 = 6}$ $3 \times 152 = 456$</p> <p>So: $458 \div 3 = 152 \text{ remainder } 2$</p>
x	200	70	5	Grid method							
8	1600	560	40								

Year	Multiplication	Division																						
5/6	<p>Example: 24×16</p> $24 \times 16 \begin{cases} 24 \times 10 = 240 \\ 24 \times 6 = (20 \times 6) + (4 \times 6) \\ = 144 \\ 240 + 144 = 384 \end{cases}$ <p>6)</p> <p>or</p> $(24 \times 20) - (24 \times 4) \\ = 480 - 96 \\ = 384$ <p>or</p> <table style="border-collapse: collapse; margin-left: 40px;"> <tr><td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 2px 10px;">20</td><td style="border-bottom: 1px solid black; padding: 2px 10px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">10</td><td style="padding: 2px 10px;">200</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">6</td><td style="padding: 2px 10px;">120</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;"></td><td style="padding: 2px 10px;">24</td></tr> </table> <p style="margin-left: 40px;">$26 \times 16 = 200 + 120 + 40 + 24 \\ = 384$</p> <p><i>Note: This is a very popular method is known as the 'box' or 'grid' method. This is an algorithmic method that does require understanding but is utilised very effectively by many pupils.</i></p>	20	4	10	200	6	120		24	<p>Example: $432 \div 15$</p> <table style="margin-left: 40px;"> <tr><td style="text-align: right;"><u>20</u> + 8</td><td></td></tr> <tr><td style="text-align: right;">15) 432</td><td>$15 \times 10 = 150$</td></tr> <tr><td style="text-align: right;">(15 x 20) <u>300</u></td><td>$15 \times 20 = 300$</td></tr> <tr><td style="text-align: right;">132</td><td>$15 \times 4 = 60$</td></tr> <tr><td style="text-align: right;">(15 x 8) <u>120</u></td><td>$15 \times 8 = 120$</td></tr> </table> <p>$\therefore 432 \div 15 = 28 \text{ remainder } 12$</p> <p>The above method is referred to as 'chunking', as they are subtracting chunks of 15 at a time.</p>	<u>20</u> + 8		15) 432	$15 \times 10 = 150$	(15 x 20) <u>300</u>	$15 \times 20 = 300$	132	$15 \times 4 = 60$	(15 x 8) <u>120</u>	$15 \times 8 = 120$				
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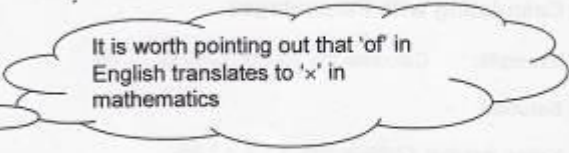
Many lower attaining pupils experience great difficulty in understanding some formal methods of calculation. It is essential to build upon what the child knows, understands and can do. The examples above are real examples of how pupils have approached the problems.

Calculation with fractions, decimals and percentages

NON-CALCULATOR METHODS

Calculating with fractions

Example: Calculate $\frac{5}{6}$ of £48



It is worth pointing out that 'of' in English translates to 'x' in mathematics

Solution:

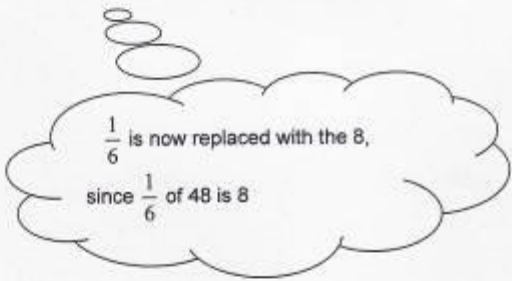
Now $\frac{5}{6} = 5 \times \frac{1}{6}$, but before we can find $\frac{5}{6}$, we need to find the value of $\frac{1}{6}$, then multiply by 5.

To find $\frac{1}{6}$ of 48 we need to divide 48 by 6.

So, $\frac{1}{6}$ of 48 = $48 \div 6 = 8$

We can now replace $\frac{1}{6}$ with 8.

So, $\frac{5}{6}$ of 48 = $5 \times \frac{1}{6} = 5 \times 8 = 40$.



$\frac{1}{6}$ is now replaced with the 8,
since $\frac{1}{6}$ of 48 is 8

Final answer

$$\frac{5}{6} \text{ of } \text{£}48 = \text{£}40$$

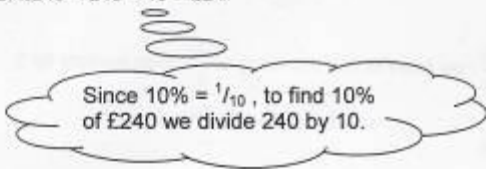
Calculating with Percentages

Example: Calculate the VAT on £240 @ 17.5%

Solution:

Noting first that $17.5\% = 10\% + 5\% + 2.5\%$

Now 10% of £240 = $240 \div 10 = £24$



Since $10\% = \frac{1}{10}$, to find 10% of £240 we divide 240 by 10.

Now, since 10% is equal to **£24**

and

$\frac{1}{2}$ of 10% is 5%

then

5% of £24

= $\frac{1}{2}$ of £24

= **£12**

Similarly,

2.5% of £24 = **£6**.

We have then:

17.5% of £240

= $[10\% + 5\% + 2.5\%]$ of £240

= £24 + £12 + £6

= **£42**

Final answer:

17.5% of £240 = £42

We can of course use this method to calculate any percentage but this method is best used when the numbers are relatively simple. See the example below.

Example: Calculate 45% of £36

Solution: 45% of £36

$$= [4 \times 10\% + 5\%] \text{ of } \pounds 36$$

Now, 10% of £36

$$= 36 \div 10 = 3.6$$

$$= \pounds 3.60,$$

and similarly

$$5\% = \pounds 1.80$$

We then have

$$40\% = 4 \times 10\%$$

$$= 4 \times 3.6$$

$$= 14.4$$

$$= \pounds 14.40$$

So,

$$45\% \text{ of } \pounds 36$$

$$= [40\% + 5\%] \text{ of } \pounds 36$$

$$= \pounds 14.40 + \pounds 1.80$$

$$= \pounds 16.20$$

Final answer: 45% of £36 = £16.20

Whilst it is possible, it would be inefficient to calculate 23.608% of 406.87kg using this method.

For this a calculator is best used!

Definitely worth reinforcing that 3.6 means £3.60 and not £3.06

Calculator Methods

Calculator methods generally involve changing the fraction or percentages into a decimal first. Decimals are often referred to as "multiplying factors" since, having turned the fraction or percentage into a decimal, we use it to multiply.

Changing a fraction into a decimal

To change a fraction into a decimal, you 'divide the top number (numerator) by the bottom number (denominator)'. To help pupils to remember this it is often useful to remind them that a fraction looks like



Changing a percentage into a decimal

To change a percentage into a decimal we aim to remember that percentage means "per-hundred". Therefore 23% means 23 per 100. This is written as $\frac{23}{100}$, and to change this fraction into a decimal we divide 23 by 100, as above.

Lower ability students will still have a problem with this skill area.

Note:

Most pupils simply need to remember that to change a percentage into a decimal, they should divide the percentage by 100.

A "starter" to a lesson where this skill needs to be utilised might look like this, where the pupils need to fill in the blank spaces with the appropriate value.

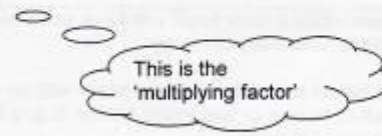
As this is a recurring decimal, how many decimal places do we need to write down? It is worth pointing out that you would use all the numbers on the calculator and round off at the end of the calculation only.

Fraction	Decimal	Percentage
$\frac{4}{5}$?	?
?	0.125	?
?	?	65%
$\frac{8}{13}$?	?

Example: Calculate 5.35% of 23,456kg

Solution: 5.35% as a decimal?

$$5.35 \div 100 = .0535$$



This is the
'multiplying factor'

So,

$$0.0535 \times 23,456 = 1,254.896$$

Final answer

$$5.35\% \text{ of } 23,456\text{kg} = 1,254.896\text{kg}$$

Depending upon the context, a suitable degree of accuracy would now be required.

Numeracy Skill by Year group

The following are the basic skills a core pupil will have achieved by the end of the given year – as outlined in the National Numeracy Strategy.

Pupils working at a level below their current year peers will be expected to, in most cases, be working at a level equivalent to one or two years below. E.g. a low ability year 7 will be working at an equivalent year 5/6 pupil.

Years 5 and 6 Skills

- Use all four operations to solve simple word problems involving number and quantities based on real life money and measures
- Multiply and divide positive whole numbers by 10, 100 and 1000
- Order positive and negative numbers
- Use decimal and negative numbers
- Use decimal notation for tenths and hundredths
- Find basic fractions of quantities, e.g. $\frac{1}{2}$ of, $\frac{1}{4}$ of, $\frac{1}{10}$ of
- Find simple percentages of quantities, e.g. 10%, 50%, 25%
- Basic calculator skills but NO scientific calculator skills taught at primary school
- Round a number with one or two decimal places to the nearest whole number
- Use, read and write standard metric units, including their abbreviations, and conversions between them
- Suggest suitable units and measuring equipment to estimate or measure length, mass or capacity
- Measure and draw lines to the nearest millimetre
- Calculate area and perimeter of simple shapes
- Record estimates and readings from scales to a suitable degree of accuracy
- Recognise perpendicular and parallel lines
- Recognise where a shape will be after reflection
- Read and plot co-ordinates in all four quadrants
- Use a protractor to measure angles **up to 180°**
- Find the mode and range of a set of data and begin to calculate a mean of a set of data
- Able to extract and **interpret** data in tables and bar charts

Year 7 Skills

All of the above, plus ...

- Know and use the order of operations, including brackets, ie brackets, \times and \div , then $+$ and $-$
- Mental methods to calculate simple decimals, fractions and percentages
- Check a result by considering whether it is of the right magnitude – estimation
- Use a letter to represent unknown numbers
- Simplify algebraic expressions, e.g. $x + 2y + 3x - y = y + 4x$
- Use simple formulae
- Calculate area, perimeter and volume for simple shapes
- Design a data collection sheet or questionnaire to use in a simple survey; construct frequency tables
- Construct, on paper and using ICT, graphs and diagrams to represent data including bar-line graphs, frequency diagrams
- Use ICT to generate pie charts
- Compare data using the range, mean, median and mode
- Interpret diagrams and graphs (including pie charts) and draw simple conclusions based on the shape of graphs and simple statistics

Year 8 Skills

All of the above plus ...

- Add, subtract, multiply and divide positive and negative numbers
- Calculate any fraction of any quantity
- Use squares, square roots, cubes and cube roots
- Find any percentage of any quantity
- Reduce a ratio to its simplest form
- Use a scientific calculator to carry out more difficult calculations
- Solving simple algebraic equations, e.g. $2x + 3 = 15$
- Plot graphs of simple algebraic equations, including interpretation of the gradient of a straight line
- Use and read and write standard metric units to solve problems involving perimeter, area and volume
- Calculate area, perimeter and volume of simple shapes including triangles

- Understand and use the language and notation associated with enlargement
- Collect data using a suitable method, including data logging using ICT
- Able to construct, extract and interpret data in tables, bar charts, pie charts, simple line graphs and scatter graphs (**NB – both on paper and using ICT**)
- Find the mean, median, mode and range of a set of data, both discrete and continuous, and begin to calculate mean of a set of continuous data

Year 9 Skills

All of the above plus ...

- Begin to use numbers in standard form
 - Add, subtract, multiply and divide fractions
 - Make and justify estimates and approximations of calculations
 - Use formulae from mathematics and other subjects including substitution of numbers into those formulae and changing the subject of those formulae
 - Construct graphs arising from real life problems, for example: distance time graphs, conversion graphs, e.g. temperature, currency
 - Circle facts - formulae for perimeter and area
 - Use and interpret maps and scale drawings
- *Suggest a problem to explore using statistical methods, frame questions and raise conjectures/hypotheses